GAIA - Multidisciplinary Academic Journal – ISSN 3079-6946
From Gaia College Academy of Applied Sciences & Technology
GAIA 1(1 – the Educational Spectrum)

Shwartz, H & Fuchs, A. (2025). The Cognitive Distinctions between Mathematical Problem-solving and Problem-posing Processes. *Gaia, 1*(1 – the Educational Spectrum), 37 - 50

The Cognitive Distinctions between Mathematical Problem-solving and Problem-posing Processes

Hagar Swartz

Ariel Fuchs

Gaia College – Academy of Applied Science & Technology, Israel

Abstract:

This meta-analysis explores the cognitive distinctions between mathematical problem-solving and problem-posing processes. While problem-solving primarily engages visuospatial working memory, fact retrieval, and procedural execution centered in the left inferior frontal gyrus, problem-posing activates distinct neural pathways involving metacognition, conceptual integration, and dorsolateral prefrontal networks. Our synthesis of neuroimaging, behavioral, and educational research reveals that these complementary processes influence cognitive arousal, self-efficacy, and motivation in mathematics education through different mechanisms. Problem-solving effectiveness correlates with spatial working memory capacity and cognitive flexibility, while successful problem-posing depends on metacognitive monitoring and language processing abilities. These findings suggest that pedagogical approaches should intentionally develop both skill sets through targeted cognitive training that addresses working memory constraints while fostering reflective thinking. Educational implications include the importance of integrating question-creation activities alongside traditional problem-solving to optimize mathematical reasoning abilities and enhance student engagement.

Keywords

Mathematical cognition, problem-posing, visuospatial working memory, metacognition, neural networks, cognitive flexibility, mathematics education, cognitive arousal, self-efficacy, motivation.

Received: 13 February 2025
First revision: 25 February 2025
Accepted: 13 Merch 2025

Introduction

The cognitive architecture underlying mathematical reasoning involves distinct yet interrelated neural and psychological processes depending on task demands. While extensive research has focused on mechanisms supporting problem solving, recent studies reveal that problem posing—the creation of novel mathematical questions—engages qualitatively different cognitive operations. This report synthesizes evidence from neuroimaging, behavioral experiments, and educational research to contrast the cognitive mechanisms activated during mathematical problem solving versus problem posing. Key findings indicate that problem solving prioritizes working memory-dependent retrieval and automated fact access, whereas problem posing relies on hierarchical goal setting, conceptual integration, and metacognitive monitoring. Neurocognitive divergences emerge in prefrontal cortex engagement patterns: solving tasks activate left inferior frontal gyrus (IFG) regions associated with arithmetic fact retrieval (Suárez-Pellicioni, Demir-Lira & Booth, 2021), while posing tasks recruit dorsolateral prefrontal networks governing abstract reasoning and task design. Educational implications highlight the need for pedagogical strategies that separately cultivate solving efficiency and posing creativity through targeted cognitive training (Zhang, Song, Wu, & Cai, 2023). Recent studies have revealed the intricate nature of this relationship, demonstrating that parental stimulation, including a supportive home environment and positive interactions, is crucial for optimal cognitive growth (Zain & Iswinarti, 2024). The relationship appears to be bidirectional, as cognitive stimulation by parents at age 2 predicts reading ability at age 4, while children's cognitive ability also influences subsequent parenting quality (Tucker-Drob & Harden, 2012). This transactional process suggests a more complex developmental dynamic than previously understood.

Research Questions

What cognitive mechanisms operate when a person solves a mathematical question vs. the active cognitive mechanisms when writing a mathematical question?

Literature Review

Foundational Cognitive Processes in Mathematical Reasoning

Mathematical cognition operates through dynamic interactions between core number representation systems, memory networks, and executive control mechanisms. The triple-code theory posits three neuroanatomical circuits: a verbal system for arithmetic facts (left perisylvian areas), a quantitative magnitude system (bilateral intraparietal sulci), and a visuospatial system for complex calculation (right parietal regions. These systems interface with prefrontal cortical regions responsible for working memory, cognitive flexibility, and goal maintenance—functions critical for solving and posing tasks but utilized differently across contexts, demonstrating the adaptability of cognitive functions in mathematical reasoning (Cai et al., 2023; Zhang, Song, Wu, & Cai, 2023).

The cognitive mechanisms underlying mathematical problem-solving

Working memory capacity differentially constrains solving versus posing performance. During problem solving, spatial working memory facilitates retention of intermediate calculation steps and visual-spatial number representations (Cai et al., 2023). For example, maintaining place value during multi-digit subtraction requires continuous updating of spatial working memory buffers (Geary et al., 2007) . In contrast, problem posing demands sustained activation of semantic memory networks to evaluate conceptual coherence and contextual appropriateness of candidate questions (Zhang, Song, Wu, & Cai, 2023). The Wisconsin Card Sorting Test reveals that solving open-ended problems requires reactive cognitive flexibility—the ability to shift problem-solving strategies based on feedback—while closed problems depend on procedural memory consolidation (Cai et al., 2023).

Longitudinal fMRI data demonstrates that positive math attitudes enhance left IFG activation during fact retrieval, suggesting motivational factors modulate working memory efficiency in solving tasks (Krueger et al., 2008). This effect operates through increased cognitive effort expenditure: students with growth mindsets persist longer in retrieving partially encoded facts rather than defaulting to compensatory counting strategies. Recent studies have illuminated the neurocognitive mechanisms underlying the relationship between math attitudes, achievement, and brain activation. Positive math attitudes are associated with increased activation in the left inferior frontal gyrus (IFG) during arithmetic problem-solving, particularly for children with lower math skills (Demir-Lira et al., 2019; Suárez-Pellicioni et al., 2021). This enhanced IFG activation reflects greater effort in fact retrieval and is linked to improved multiplication skills over time (Suárez-Pellicioni et al., 2021). Additionally, positive attitudes correlate with increased hippocampal engagement, supporting more efficient memory-based strategies (Chen et al., 2018). The left angular gyrus is specifically involved in arithmetic fact retrieval (Grabner et al., 2009), while temporal cortex activation explains improvements in math attitudes. Growth mindset fosters cognitive development through enhanced cortico-striatal dynamics. Longitudinally, motivation and learning strategies, rather than intelligence, predict growth in math achievement (Suárez-Pellicioni & Booth, 2022).

The cognitive mechanisms underlying mathematical problem-solving reveal a complex interplay of mental processes, with working memory (WM) playing a central role. Research demonstrates that spatial working memory is crucial for numerical understanding and geometry (Silverman & Ashkenazi, 2022), with visuospatial WM specifically supporting spatial numerical representations and decomposition strategies. Studies have consistently shown that visuospatial working memory is essential for solving planning tasks, such as the Tower of London (Gilhooly et al., 2002) and the Tower of Hanoi (Cushen & Wiley, 2011), while also facilitating mental arithmetic, especially when employing counting strategies (Hubber, 2015).

Neural network engagement during mathematical problem-solving activates a bilateral network encompassing prefrontal, parietal, and inferior temporal regions (Amalric & Dehaene, 2016). These neural patterns exhibit distinct characteristics during different phases of mathematical engagement, including instruction, problem-solving, and example-based learning (Lee et al., 2015). The activation patterns suggest a sophisticated neural architecture that supports the execution of established mathematical skills and the acquisition of new competencies (Wintermute et al., 2012).

Cognitive flexibility emerges as a critical mechanism, particularly in solving open-ended mathematical problems. This flexibility enables strategy adaptation and the generation of multiple solution pathways, influenced by cognitive style, creativity, and verbal ability (Bahar & Ozturk, 2018). Research indicates that traditional mathematical instruction may inadvertently constrain the development of this flexibility, as evidenced by college students' limited ability to generate multiple strategies (Shaw et al., 2022).

The integration of different types of mathematical knowledge represents another crucial cognitive mechanism. The relationship between conceptual knowledge (understanding core concepts) and procedural knowledge (executing solution steps) appears to be iterative, with each form of knowledge potentially enhancing the other (Rittle-Johnson, 2004). Prior knowledge significantly affects problem encoding and solution processes (Crooks & Alibali, 2013), while mathematical symbols can prime specific procedural responses (Fayol & Thevenot, 2012).

Cognitive load management varies based on task difficulty, prior knowledge, and interface familiarity (Oviatt et al., 2006). Research demonstrates that different instructional strategies are optimal at different stages of mathematical development, with worked examples proving more effective for novices while direct problem-solving becomes superior for advanced learners (Atkinson, Renkl, & Merrill, 2003).

These findings have significant implications for mathematics education, particularly concerning learning disabilities. Studies consistently show that mathematics learning disabilities often manifest as impairments in spatial working memory and difficulties with visuospatial tasks (Passolunghi & Mammarella, 2012). Effective interventions, therefore, should incorporate spatial cognition training, visuospatial working memory exercises, and careful comparison of worked examples.

The research synthesis suggests that successful mathematical problem-solving relies on the coordinated operation of multiple cognitive mechanisms, with spatial working memory and cognitive flexibility playing particularly crucial roles. This understanding indicates that effective mathematics education should develop both spatial cognition and cognitive flexibility while carefully managing cognitive load based on learner expertise. Future research might productively explore the optimal sequencing of different types of mathematical activities to maximize the development of these various cognitive mechanisms.

The research on cognitive mechanisms in mathematical problem-solving and writing reveals a fascinating interplay between working memory, metacognition, and language processing. Working memory serves as a fundamental cognitive resource, with visuo-spatial working memory being particularly crucial for mathematical operations. Studies have demonstrated its essential role in planning tasks such as the Tower of London (Gilhooly et al., 2002) and the Tower of Hanoi (Cushen & Wiley, 2011), as well as in mental arithmetic and counting strategies (Hubber, 2015).

The relationship between working memory and mathematical ability is particularly evident in studies of learning disabilities, where children with mathematics learning disabilities consistently show impairments in spatial working memory tasks (Passolunghi & Mammarella, 2012). This connection extends to everyday problem-solving tasks, including spatial text processing and visual search activities (Travis, 2019), highlighting the pervasive role of working memory in mathematical cognition.

Writing in mathematics emerges as a powerful tool for developing metacognitive skills and enhancing problem-solving abilities. Research by Pugalee (2001) and Kosko et al. (2014) demonstrates that writing promotes higher-order thinking by engaging students in reflection, anticipation of solution paths, and evaluation of question complexity. This process helps students organize and consolidate their mathematical thinking while developing their ability to communicate ideas coherently (Goldsby & Cozza, 2002).

The development of algebraic reasoning and abstraction is particularly supported through writing activities (Kosko et al., 2014). Journal writing, as identified by Waywood (1992), serves as a valuable tool for concept formation in mathematics. This finding aligns with research showing that writing helps students make connections between abstract mathematical concepts and real-world contexts (Gainsburg, 2008).

Language factors play a significant role in mathematics learning, as demonstrated by Aiken's (1972) research on the influence of reading ability and verbal skills. The use of questions in mathematical writing serves multiple instructional and conversational functions (Mackiewicz & Thompson, 2014), suggesting that developing question-writing skills can enhance both teaching and learning processes.

Teachers can facilitate these cognitive processes by modeling thinking strategies, providing structured problem-solving opportunities, and helping students develop metacognitive awareness. The development of mathematical literacy question writing skills in preservice teachers has been shown to improve their awareness and instructional capabilities (Gartmann & Freiberg, 1995).

This synthesis of research suggests that effective mathematics education should integrate writing activities that promote metacognition while considering the constraints and capabilities of working memory. The combination of writing tasks with appropriate cognitive support mechanisms may offer a powerful approach to developing mathematical understanding and problem-solving abilities.

The findings indicate a need for further research into how different types of writing tasks might be optimized to support various aspects of mathematical learning while managing cognitive load. Additionally, investigating the relationship between working memory capacity and the effectiveness of different writing strategies could provide valuable insights for educational practice.

Findings

The cognitive mechanisms involved in solving a mathematical question versus writing a mathematical question differ in several fundamental ways, particularly in terms of working memory utilization, metacognitive engagement, and neural network activation.

Cognitive Mechanisms in Mathematical Problem-Solving

When a person solves a mathematical problem, cognitive engagement primarily relies on working memory, particularly visuospatial working memory (WM), which supports numerical reasoning, spatial visualization, and procedural fluency (Silverman & Ashkenazi, 2022). Research shows that individuals performing mathematical operations engage bilateral prefrontal and parietal networks, essential for both arithmetic processing and logical reasoning (Amalric & Dehaene, 2016).

Another key cognitive process in problem-solving is cognitive flexibility, which allows individuals to adapt strategies and explore multiple solution pathways (Star & Rittle-Johnson, 2008). This flexibility is particularly necessary for open-ended problems, where solution strategies are not immediately obvious (Bahar & Ozturk, 2018). However, traditional mathematics instruction tends to emphasize rigid procedural execution, which can limit problem-solving adaptability (Shaw, 2022).

Additionally, conceptual and procedural knowledge integration plays a critical role in solving mathematical problems. Conceptual understanding (knowing why a mathematical principle works) enhances procedural efficiency, and vice versa (Rittle-Johnson, 2004). Successful problem solvers actively retrieve prior knowledge, decompose problems into manageable steps, and monitor cognitive load (Oviatt et al., 2006), which varies based on the complexity of the problem and the solver's expertise.

Mathematical disabilities often stem from deficits in spatial working memory and visuospatial processing (Passolunghi & Mammarella, 2012), reinforcing the importance of these cognitive resources in problem-solving. For instance, impairments in spatial working memory negatively impact arithmetic reasoning, counting strategies, and geometric problem-solving (Hubber, 2015).

Cognitive Mechanisms in Mathematical Writing

Writing a mathematical question, in contrast, engages metacognition, language processing, and conceptual reasoning. Unlike problem-solving, which primarily activates procedural memory and numerical cognition, question-writing demands reflective thinking and linguistic structuring (Kosko et al., 2014).

Studies indicate that writing about mathematics strengthens higher-order cognitive functions, such as concept organization, abstraction, and anticipation of problem-solving pathways (Pugalee, 2001). When individuals create mathematical questions, they engage in a process of deconstructing mathematical concepts, which enhances self-regulation and deep learning (Goldsby & Cozza, 2002).

Moreover, writing mathematics-related content promotes mathematical literacy, requiring individuals to articulate ideas clearly and align them with mathematical principles (Waywood, 1992). This process fosters an understanding of abstract mathematical concepts by bridging numerical operations with linguistic expressions (Dreyfus et al., 2020).

Additionally, language factors significantly impact mathematical writing. As Aiken (1972) found, verbal ability and reading comprehension influence a student's capacity to formulate mathematically coherent questions. Similarly, writing mathematical problems necessitates an awareness of instructional and conversational functions, suggesting that the ability to craft meaningful questions is linked to teaching effectiveness and knowledge transfer (Mackiewicz & Thompson, 2014).

Discussion - Comparing the Two Cognitive Processes

While both mathematical problem-solving and question-writing engage cognitive resources, they differ in their primary mechanisms.

Implications for Mathematics Education

These cognitive differences suggest that mathematics education should integrate both problem-solving and question-writing activities to optimize learning outcomes. While problem-solving develops procedural fluency and spatial reasoning, writing enhances metacognition and conceptual understanding (Kosko et al., 2014).

Teachers can leverage these cognitive mechanisms by:

1. Encouraging students to write their own mathematical questions, which promotes deeper understanding and conceptual mastery.
2. Incorporating problem-solving activities with reflective writing, reinforcing metacognitive skills.
3. Providing structured opportunities for cognitive flexibility development, allowing students to explore multiple problem-solving approaches (Huiyan et al., 2023).
4. Addressing working memory constraints in instruction, particularly for students with learning disabilities (Passolunghi & Mammarella, 2012).

References

Aiken, L. R. (1972). Research on attitudes toward mathematics. *The Arithmetic Teacher, 19*(3), 229-234.

Amalric, M., & Dehaene, S. (2016). Origins of the brain networks for advanced mathematics in expert mathematicians. *Proceedings of the National Academy of Sciences, 113*(18), 4909-4917.

Atkinson, R. K., Renkl, A., & Merrill, M. M. (2003). Transitioning from studying examples to solving problems: Effects of self-explanation prompts and fading worked-out steps. *Journal of Educational Psychology, 95*(4), 774.

Bahar, A., & Ozturk, M. A. (2018). An Exploratory Study on the Relationship between Creativity and Processing Speed for Gifted Children. *International Education Studies, 11*(3), 77-91.

Chen, L., Bae, S. R., Battista, C., Qin, S., Chen, T., Evans, T. M., & Menon, V. (2018). Positive attitude toward math supports early academic success: Behavioral evidence and neurocognitive mechanisms. *Psychological Science, 29*(3), 390-402.

Crooks, N. M., & Alibali, M. W. (2013). Noticing relevant problem features: Activating prior knowledge affects problem solving by guiding encoding. *Frontiers in Psychology, 4*, 884.

Cui, J., Wang, S., Lv, L., Ran, X., Cui, Z., & Zhou, X. (2023). Different cognitive mechanisms used for solving open and closed math problems. *International Journal of Psychology, 58*(6), 584-593.

Cushen, P. J., & Wiley, J. (2011). Aha! Voila! Eureka! Bilingualism and insightful problem solving. *Learning and Individual Differences, 21*(4), 458-462.

Dreyfus, T. (2020). Abstraction in mathematics education. *Encyclopedia of Mathematics Education*, 13-16.

Fayol, M., & Thevenot, C. (2012). The use of procedural knowledge in simple addition and subtraction problems. *Cognition, 123*(3), 392-403.

Gainsburg, J. (2008). Real-world connections in secondary mathematics teaching. *Journal of Mathematics Teacher Education, 11*, 199-219.

Gartmann, S., & Freiberg, M. (1995). Metacognition and mathematical problem solving: Helping students to ask the right questions. *The Mathematics Educator, 6*(1).

Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. Child development, 78(4), 1343-1359.

Gilhooly, K. J., Wynn, V., Phillips, L. H., Logie, R. H., & Sala, S. D. (2002). Visuo-spatial and verbal working memory in the five-disc Tower of London task: An individual differences approach. *Thinking & Reasoning, 8*(3), 165-178.

Goldsby, D. S., & Cozza, B. (2002). Writing samples to understand mathematical thinking. *Mathematics Teaching in the Middle School, 7*(9), 517-520.

Grabner, R. H., Ansari, D., Koschutnig, K., Reishofer, G., Ebner, F., & Neuper, C. (2009). To retrieve or to calculate? Left angular gyrus mediates the retrieval of arithmetic facts during problem solving. *Neuropsychologia, 47*(2), 604-608.

Hubber, P. J. (2015). *Understanding the role of visuo-spatial working memory in adult mathematics*. (Doctoral dissertation, University of Nottingham).

Huiyan, Y. E., Oi-Lam, N. G., & Zhihao, C. U. I. (2023). A Thematic Analysis Exploring Flexibility in Programming-based Mathematical Problem Solving. In *International Conference on Computers in Education.*

Kosko, K. W., Rougee, A., & Herbst, P. (2014). What actions do teachers envision when asked to facilitate mathematical argumentation in the classroom?. *Mathematics Education Research Journal, 26*, 459-476.

Krueger, F., Spampinato, M.V., Pardini, M., Pajevic, S., Wood, J.N., Weiss, G.H., Landgraf, S., & Grafman, J.H. (2008). Integral calculus problem solving: an fMRI investigation. *Neuro Report, 19*, 1095-1099.

Lee, H. S., Fincham, J. M., & Anderson, J. R. (2015). Learning from examples versus verbal directions in mathematical problem solving. *Mind, Brain, and Education, 9*(4), 232-245.

Mackiewicz, J., & Thompson, I. (2014). Instruction, cognitive scaffolding, and motivational scaffolding in writing center tutoring. *Composition Studies, 42*(1), 54-78.

Oviatt, S., Arthur, A., & Cohen, J. (2006). Quiet interfaces that help students think. In *Proceedings of the 19th annual ACM symposium on User interface software and technology*, 191-200.

Passolunghi, M. C., & Mammarella, I. C. (2012). Selective spatial working memory impairment in a group of children with mathematics learning disabilities and poor problem-solving skills. *Journal of learning disabilities, 45*(4), 341-350.

Pugalee, D. K. (2001). Writing, mathematics, and metacognition: Looking for connections through students' work in mathematical problem solving. *School science and mathematics, 101*(5), 236-245.

Rittle-Johnson, B. (2004). Promoting flexible problem solving: The effects of direct instruction and self-explaining. In *Proceedings of the Annual Meeting of the Cognitive Science Society*, 26.

Shaw, M. A. (2022). *Teacher Perspective of the Educational Impact when Transitioning to a Virtual Setting During a Pandemic* (Doctoral dissertation, Northcentral University).

Silverman, S., & Ashkenazi, S. (2022). The unique role of spatial working memory for mathematics performance. *Journal of Numerical Cognition, 8*(1), 226-243.

Star, J. R., & Rittle-Johnson, B. (2008). Flexibility in problem solving: The case of equation solving. *Learning and Instruction, 18*(6), 565-579.

Suárez-Pellicioni, M., Demir-Lira, Ö. E., & Booth, J. R. (2021). Neurocognitive mechanisms explaining the role of math attitudes in predicting children's improvement in multiplication skill. *Cognitive, Affective, & Behavioral Neuroscience, 21*, 917-935.

Suárez-Pellicioni, M., & Booth, J. R. (2022). Temporal cortex activation explains children's improvement in math attitudes. *Child Development, 93*(4), 1012-1029.

Travis, C. (2019). Spatial humanities GIS: The city as a literary, historical, and cultural STEAM lifeworld laboratory. In *The STEAM revolution: Transdisciplinary approaches to science, technology, engineering, arts, humanities and mathematics*, 13-31.

Waywood, A. (1992). Journal writing and learning mathematics. *For the Learning of Mathematics, 12*(2), 34-43.

Wintermute, S., Betts, S., Ferris, J. L., Fincham, J. M., & Anderson, J. R. (2012). Brain networks supporting execution of mathematical skills versus acquisition of new mathematical competence. *PLoS One, 7*(12), e50154.

Zhang, L., Song, N., Wu, G., & Cai, J. (2023). Understanding the cognitive processes of mathematical problem posing: evidence from eye movements. *Educational Studies in Mathematics*, 1-30.